GEORGIA INSTITUTE OF TECHNOLOGY
OFFICE OF RESEARCH ADMINISTRATION

h. m82 1-13 master now final rec

Date: 17 September 1968

RESEARCH PROJECT INITIATION

Project Title: Dynamics of Propellant in a Spinning Container and Description ~~and~~ by a Mechanical Model

Project No.: B-903

Project Director: Dr. Helmut F. Bauer

Sponsor: U. S. Army Missile Command, Redstone Arsenal, Alabama

Agreement Period: From 30 August 1968 until 30 August 1969

Type Agreement: Contract No. DAAH01-69-C-0296

Amount: $24,900

Contract and Property Administration

Mr. R. J. Whitcomb
ONR Resident Representative
103-104 Electronics Research Building
Georgia Institute of Technology

Technical Direction

U. S. Army Missile Command
Attn: AMSMI-RSD
Redstone Arsenal, Alabama 35809

Reports Required

Cost and Performance - Monthly, by 10th working day of subsequent month, 4 copies required by sponsor.

Progress - Monthly, informal letter type - by 10th working day of subsequent month, 4 copies required by sponsor.

Technical - As they occur, 22 copies required by sponsor.

Final - Not later than 30 August 1969, 22 copies required by sponsor.

Assigned to: School of Engineering Mechanics

COPIES TO:

- [X] Project Director
- [X] School Director
- [X] Dean of the College
- [X] Administrator of Research
- [X] Associate Controller (2)
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Other Mr. R. A. Martin
File B-903

RA-3 (4/65)

B-903

ENGINEERING COLLEGE

GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL OF ENGINEERING SCIENCE
AND MECHANICS

225 NORTH AVENUE, N.W.
ATLANTA, GEORGIA 30332

October 10, 1968

U. S. Army Missile Command
Redstone Arsenal, Alabama 35809

Attn: AMSMI-RSD

Subject: Monthly Progress Report # 1
Contract # DAAH01-69-C-0296 for period September 1-30, 1968
"Dynamics of Propellant in a Spinning Container and Description by a Mechanical Model."

Gentlemen:

During the subject period a literature survey has been conducted and some of the available literature has been obtained or ordered. With the available trajectory data the steady state free surface elevation has been determined for various flight times, in order to see the effect of the longitudinal acceleration and centrifugal acceleration of the missile upon the free propellant surface shape. With these results the accuracy of the approximations for the natural frequencies of the liquid can be checked.

No difficulties have been encountered.

Sincerely yours,

Helmut F. Bauer
Professor

HFB:ps

cc: AMSMI-RSD - 2
AMSMI-RSP - 1
AMSMI-IWD - 1

GEORGIA INSTITUTE OF TECHNOLOGY
School of Engineering Science and Mechanics

Monthly Cost Report Month of: September, 1968

Project Title: "Dynamics of Propellant in a Spinning Container and Description and a Mechanical Model"

Project No: B-903 Contract No: DAAH01-69-C-0296 Project Director: Dr. H. F. Bauer

Sponsor: U. S. Army Missile Command, Redstone Arsenal, Alabama

		FOR MONTH	TO DATE
Manhours Expended:	Project Director	15	15
	Graduate Assistants		
Funds Expended:			
Personal Services		$486.11	$486.11
Material and Operating Expenses			
Travel			
Computer Use Charges			
Overhead		277.08	277.08
Total Funds Expended		$763.19	$763.19
Percentage of Work Completed:		2%	2%

NOTE: Although fall quarter began September 23 and there was only one week's work involved, the Georgia Tech accounting system uses four salary checks for each quarter. Hence, the hours and salary for this month are out of

Project Director

ENGINEERING COLLEGE

GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL OF ENGINEERING SCIENCE
AND MECHANICS

225 NORTH AVENUE, N.W.
ATLANTA, GEORGIA 30332

November 6, 1968

U.S. Army Missile Command
Redstone Arsenal, Alabama 35809

Attn: AMSMI-RSD

Subject: Monthly Progress Report #2
Contract # DAAH01-69-C-0296 for period October 1-31, 1968
"Dynamics of Propellant in a Spinning Container and Description by a Mechanical Model."

Gentlemen:

During the subject period the hydrostatic problem of a spinning annular cylinder with a bottom and top has been completely solved for the various liquidheights and angular frequency ranges. It was found that there are two basic liquidheight ranges, $h < H/2$ and $h \geq H/2$, h being liquid height and H being the container height, and that there also are three basic ranges for the angular velocity Ω_o to each of these liquidheight ranges. The free liquid surface elevation was derived for each of these ranges. Also, the steady state free liquid surface elevation for some flight times of the missile at consideration had been completed.

Furthermore the problem of forced oscillations of the rotating liquid in an annular container for small angular velocity has been formulated.

Difficulties shall occur in the determination of the natural frequencies of the liquid, if Ω_o is not considered small.

Sincerely yours,

Helmut F. Bauer, Professor

HFB:jv

cc: AMSMI-RSD - 2
AMSMI-RSP - 1
AMSMI-IWD - 1

GEORGIA INSTITUTE OF TECHNOLOGY
School of Engineering Science and Mechanics

Monthly Cost and Performance Report # 2 Month of: October, 1968

Project Title: "Dynamics of Propellant in a Spinning Container and Description and a Mechanical Model"

Project No: B-903 Contract No: DAAH01-69-C-0296 Project Director: Dr. H. F. Bauer

Sponsor: U. S. Army Missile Command, Redstone Arsenal, Alabama

		FOR MONTH	TO DATE
Manhours Expended:	Project Director	58	73
	Graduate Assistants	58	58
Funds Expended:			
Personal Services		$721.11	$1,207.22
Material and Operating Expenses			
Travel			
Computer Use Charges			
Overhead		411.03	688.12
Total Funds Expended		$1,132.14	$1,895.34
Percentage of Work Completed:		6%	8%

Project Director

ENGINEERING COLLEGE

GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL OF ENGINEERING SCIENCE
AND MECHANICS

225 NORTH AVENUE, N.W.
ATLANTA, GEORGIA 30332

December 9, 1968

U.S. Army Missile Command
Redstone Arsenal, Alabama 35809

Attention: AMSMI-RSD

Subject: Monthly Progress Report #3
Contract # DAAH01-69-C-0296 for period November 1-31, 1968
"Dynamics of Propellant in a Spinning Container and Description by a Mechanical Model."

Gentlemen:

During the subject period the free oscillation and natural frequencies of the liquid in the spinning container have been treated for the case I and III, i.e. the case where the ratio $\Omega_0^2/2g$ is large enough, that the undisturbed free liquid surface is at $r \approx \tau$. The liquid elevation above the undisturbed free surface, the pressure-distribution, the liquid force in x-direction and liquid moment about the y-axis have been analytically obtained for the free oscillation case.

On November 21, 1968 a meeting was held at the U. S. Army Missile Command, Redstone, Alabama at Mr. Goessling's office, where the progress of the project has been reported. Due to the fact of malorientation of propellant at reduced thrust, it was recommended to place a baffle of proper width at a proper height in the container to avoid the migration of the monies portion of the remaining propellant to the forward bulkhead of the tank. Theoretical investigations are in progress.

It was agreed upon that most of the lengthy computation of the problem of sloshing in spinning containers could be performed at the U. S. Army Missile Command.

Sincerely yours,

Helmut F. Bauer, Professor

HFB:jv

cc: AMSMI-RSD - 2
AMSMI-RSP - 1
AMSMI-IWD - 1

REVISED

GEORGIA INSTITUTE OF TECHNOLOGY
School of Engineering Science and Mechanics

Monthly Cost and Performance Report # 2 Month of: October, 1968

Project Title: "Dynamics of Propellant in a Spinning Container and Description and a Mechanical Model"

Project No: B-903 Contract No: DAAH01-69-C-0296 Project Director: Dr. H. F. Bauer

Sponsor: U.S. Army Missile Command, Redstone Arsenal, Alabama

		FOR MONTH	TO DATE
Manhours Expended:	Project Director	58	73
	Graduate Assistants	116	116
Funds Expended:			
Personal Services		$ 956.11	$1,442.22
Material and Operating Expenses			
Travel			
Computer Use Charges			
Overhead		544.98	822.06
Total Funds Expended		$1,501.09	$2,244.28
Percentage of Work Completed:		6%	8%

GEORGIA INSTITUTE OF TECHNOLOGY
School of Engineering Science and Mechanics

Monthly Cost and Performance Report # 3 Month of: November, 1968

Project Title: "Dynamics of Propellant in a Spinning Container and Description and a Mechanical Model"

Project No: B-903 Contract No: DAAH01-69-C-0296 Project Director: Dr. H. F. Bauer

Sponsor: U.S. Army Missile Command, Redstone Arsenal, Alabama

		FOR MONTH	TO DATE
Manhours Expended:	Project Director	58	131
	Graduate Assistants	116	232
Funds Expended:			
Personal Services		$ 956.11	$2,378.33
Material and Operating Expenses			
Travel			
Computer Use Charges			
Overhead (57% of Personal Services)		544.98	1,367.04
Total Funds Expended		$1,501.09	$3,745.38
Percentage of Work Completed:		6%	14%

Project Director

xrox

ENGINEERING COLLEGE

GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL OF ENGINEERING SCIENCE
AND MECHANICS

225 NORTH AVENUE, N.W.
ATLANTA, GEORGIA 30332

January 9, 1969

B-903

U.S. Army Missile Command
Redstone Arsenal, Alabama 35809

Attention: AMSMI-RSD

Subject: Monthly Progress Report #4
Contract #DAAH01-69-C-0296 for period December 1-31, 1968
"Dynamics of Propellant in a Spinning Container and Description by a Mechanical Model."

Gentlemen:

During the subject period the liquid surface elevation in a container with a conical top was investigated. This was performed in view of the fact that a conical baffle may be introduced into the tank in order to avoid malorientation of the propellant at low and negative longitudinal acceleration. The free surface elevation for such a system was also determined for various Ω_o-values.

Furthermore an approximate free surface condition for the case where the surface elevation is such, that the free liquid surface condition can be neither satisfied at $z \approx h$ for $\Omega_o^2/2g \ll 1$ nor for $r = c$ for very large Ω_o-values, has been formulated.

Sincerely yours,

Helmut F. Bauer, Professor

HFB:jv

cc: AMSMI-RSD - 2
AMSMI-RSP - 1
AMSMI-IWD - 1

ENGINEERING COLLEGE

GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL OF ENGINEERING SCIENCE
AND MECHANICS

225 NORTH AVENUE, N.W.
ATLANTA, GEORGIA 30332

February 4, 1969

U.S. Army Missile Command
Redstone Arsenal, Alabama 35809

Attention: AMSMI-RSD

Subject: Monthly Progress Report #5
Contract #DAAH01-69-C-0296 for period January 1-31, 1969
"Dynamics of Propellant in a Spinning Container and Description by a Mechanical Model."

Gentlemen:

During the subject period the free oscillation case has been completed for region I and III. The equations for the Eigenvalues have been derived and presented to the Computation Laboratory of the U.S. Army Missile Command Redstone Arsenal.

Furthermore the liquid forces and moments for the elliptic and hyperbolic cases for region I and II have been determined.

Some thought has been given to the representation of the liquid motion by a mechanical model. On January 30 and 31, 1968 a meeting was held at the U.S. Army Missile Command, Redstone Arsenal, Alabama at Mr. Goessling's office, where the progress of the project has been reported. Present at the meeting were:

H. F. Bauer, School of Engineering Science and Mechanics, Ga. Tech
F. Goessling, U. S. Army Missile Command
W. Nash, U. S. Army Missile Command
J. Sofferis, U. S. Army Missile Command
W. Martin, U. S. Army Missile Command
D. H. Dahlene, U. S. Army Missile Command

The programming of the determinants of Bessel Functions and modified Bessel-functions for the determination of the roots ξ_{mn} and the natural frequency of the liquid in the spinning container has been discussed for the various cases and regions. Furthermore the progress of the project was reported, where also the location of the conical baffle and its width was presented, in order to have during the drastic acceleration change a certain amount of fuel in a proper location.

The test-model was inspected and its transportation to the School of Engineering Science and Mechanics of Georgia Institute of Technology was discussed and agreed upon.

Analytical Difficulties that occur in the treatment of forced fuel oscillations have been explained.

Sincerely yours,

H. F. Bauer
Professor

HFB:jv

cc: AMSMI-RSD - 2
AMSMI-RSP - 1
AMSMI-IWD - 1

GEORGIA INSTITUTE OF TECHNOLOGY
School of Engineering Science and Mechanics

Monthly Cost and Performance Report # 5 . Month of: January, 1969

Project Title: "Dynamics of Propellant in a Spinning Container and Description and a Mechanical Model"

Project No: B-903 Contract No: DAAH01-69-C-0296 Project Director: Dr. H. F. Bauer

Sponsor: U.S. Army Missile Command, Redstone Arsenal, Alabama

		FOR MONTH	TO DATE
Manhours Expended:	Project Director	58	247
	Graduate Assistants	116	464
Funds Expended:			
Personal Services		$ 985.74	$4,349.79
Material and Operating Expenses			
Travel			
Computer Use Charges			
Overhead		561.86	2,490.76
Total Funds Expended		$1,547.60	$6,840.55
Percentage of Work Completed:		10%	30%

Project Director

ENGINEERING COLLEGE

GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL OF ENGINEERING SCIENCE
AND MECHANICS

225 NORTH AVENUE, N.W.
ATLANTA, GEORGIA 30332

March 5, 1969

U.S. Army Missile Command
Redstone Arsenal, Alabama 35809

Attention: SMI-RSD

Subject: Monthly Progress Report #6
Contract #DAAH01-69-C-0296 for period February 1-28, 1969
"Dynamics of Propellant in a Spinning Container and Description by a Mechanical Model."

Gentlemen:

During the subject period the spin-up problem has been investigated and the basic equation of motion for an infinitely long cylinder has been derived. Also the problems of liquid impact have been treated. From an experimental investigation performed by NASA and reported in NASA-TN-D-2913 entitled "Experimental Investigation of Liquid Impact in a Model Propellant Tank," the following conclusions may be reached by extrapolating the results of the experiments to a spinning container:

1. The flow pattern of the spinning liquid is such that it shall due to its initial spinning orientation travel up the walls of the container and hit the tank dome, where most of the impact shall be acting at the outer annulus of the tank dome.

2. The impact force depends upon the relative acceleration of the tank at the time of impact.

3. If the tank deceleration is greater than (g), then the impact force exhibits a smaller value than the hydrostatic load. Thus, if the tank dome and bottom are designed to withstand hydrostatic loads for the acceleration history of the missile, the loads resulting to tank deceleration should not result in excessive impact loads.

A film depicting the experimental results has been obtained from NASA and shall be shown at the next project meeting in the last week in March.

Sincerely yours,

H. F. Bauer
Professor

HFB:jv

cc: AMSMI-RSD - 2
AMSMI-RSP - 1
AMSMI-RWD - 1

GEORGIA INSTITUTE OF TECHNOLOGY
School of Engineering Science and Mechanics

Monthly Cost and Performance Report # 6 Month of: February, 1969

Project Title: "Dynamics of Propellant in a Spinning Container and Description and a Mechanical Model"

Project No: B-903 Contract No: DAAH01-69-C-0296 Project Director: Dr. H. F. Bauer

Sponsor: U. S. Army Missile Command, Redstone Arsenal, Alabama

		FOR MONTH	TO DATE
Manhours Expended:	Project Director	58	305
	Graduate Assistants	116	580
Funds Expended:			
Personal Services		$ 985.74	$5,335.53
Material and Operating Expenses			
Travel		*154.73	154.73
Computer Use Charges			
Overhead		561.86	3,052.62
Total Funds Expended		$1,702.33	$8,542.88
Percentage of Work Completed:		10%	40%

*This travel has not been reported previously, although $80 of this amount was during the month of November and $74.73 was during the month of January.

Project Director

ENGINEERING COLLEGE

GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL OF ENGINEERING SCIENCE
AND MECHANICS

225 NORTH AVENUE, N.W.
ATLANTA, GEORGIA 30332

April 4, 1969

U. S. Army Missile Command
Redstone Arsenal, Alabama 35809

Attention: AMSMI-RSD

Subject: Monthly Progress Report #7
Contract #DAAH01 for period March 1-31, 1969
"Dynamics of Propellant in a Spinning Container and Description by a Mechanical Model."

Gentlemen:

During the subject period the free oscillation case has been completed. All liquid forces, liquid moments pressure distributions and free liquid surface elevations have been completed and analytically derived. For large angular spin frequency Ω_0 no liquid force in x- and y-direction is present, while for the liquid force in longitudinal direction of the missile only the symmetric mode m=0 contributes. Some further thought has been given to the forced oscillation case. It is assumed that only the lower modes or lowest mode will considerably contribute to the dynamics of the vehicle. In the light of this the function r/a shall be expanded into the Eigenfunctions. The series shall then be truncated and the various terms shall be determined to see whether the amplified method shall yield good results.

A project meeting took place on March 27, and March 28, 1969 in Mr. Goessling's office. The computer program for the determination of the roots ξ_{mn} and $\bar{\xi}_{mn}$ has been discussed in great length. Furthermore the film depicting the experimental results of NASA, as reported in NASA-TN-D-2913, entitled "Experimental Investigation of Liquid Impact in a Model Propellant Tank" has been shown.

Sincerely yours,

Helmut F. Bauer
Professor

HFB:ps

cc: AMSMI-RSD - 2
AMSMI-RSP - 1
AMSMI-IWD - 1

GEORGIA INSTITUTE OF TECHNOLOGY
School of Engineering Science and Mechanics

Monthly Cost and Performance Report # 7   Month of: March, 1969

Project Title: "Dynamics of Propellant in a Spinning Container and Description and a Mechanical Model"

Project No: B-903   Contract No: DAAH01-69-C-0296   Project Director: Dr. H. F. Bauer

Sponsor: U. S. Army Missile Command; Redstone Arsenal, Alabama

		FOR MONTH	TO DATE
Manhours Expended:	Project Director	58	363
	Graduate Assistants	116	696
Funds Expended:			
Personal Services		$ 985.74	$ 6,321.27
Material and Operating Expenses			
Travel		62.67	217.40
Computer Use Charges			
Overhead		561.86	3,614.48
Total Funds Expended		$1,610.27	$10,153.15
Percentage of Work Completed:		8%	48%

Project Director H. B.

ENGINEERING COLLEGE

GEORGIA INSTITUTE OF TECHNOLOGY

OOL OF ENGINEERING SCIENCE
AND MECHANICS

225 NORTH AVENUE, N.W.
ATLANTA, GEORGIA 30332

May 5, 1969

U. S. Army Missile Command
Redstone Arsenal, Alabama 35809

Attention: AMSMI-RSD

Subject: Monthly Progress Report #8
Contract #DAAH01 for period April 1-30, 1969
"Dynamics of Propellant in a Spinning Container and Description by a Mechanical Model."

Gentlemen:

During the subject period the determination of the longitudinal liquid force F_z has been completed for the free oscillation case. For all spin-frequency ranges Ω_o it was found, that the liquid force in longitudinal direction exhibited values for sloshing only in the symmetric liquid mode $m = 0$.

The orthogonality of the Eigenfunctions has been proved and the forced oscillation case has been started. The function r/a can now be expanded, and the acceleration potential for an excitation of the form $x_o \cos \Omega t$ (where x_o is the amplitude of the forcing function and Ω is the circular forcing frequency) has been derived. For this investigation the free surface condition (for the forced case) had to be determined. The acceleration potential for all other spin-frequency ranges Ω_o as well as for the elliptic and hyperbolic cases shall be treated in the next period. Also pressure distribution, liquid forces and moments, as well as the liquid surface elevation shall be derived.

Sincerely yours,

Helmut F. Bauer
Professor

HFB:jv

cc: AMSMI-RSD - 2
AMSMI-RSP - 1
AMSMI-IWD - 1

GEORGIA INSTITUTE OF TECHNOLOGY
School of Engineering Science and Mechanics

Monthly Cost and Performance Report # 8   Month of: April, 1969

Project Title: "Dynamics of Propellant in a Spinning Container and Description and a Mechanical Model"

Project No: B-903   Contract No: DAAH01-69-C-0296   Project Director: Dr. H. F. Bauer

Sponsor: U. S. Army Missile Command; Redstone Arsenal, Alabama

		FOR MONTH	TO DATE
Manhours Expended:	Project Director	58	421
	Graduate Assistants	144	840
Funds Expended:			
Personal Services		$1,050.74	$ 7,372.01
Material and Operating Expenses			
Travel			217.40
Computer Use Charges			
Overhead		598.92	4,213.40
Total Funds Expended		$1,649.66	$11,802.81
Percentage of Work Completed:		6%	54%

Project Director

ENGINEERING COLLEGE

GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL OF ENGINEERING SCIENCE
AND MECHANICS

225 NORTH AVENUE, N.W.
ATLANTA, GEORGIA 30332

May 27, 1969

U. S. Army Missile Command
Redstone Arsenal, Alabama 35809

Attention: AMSMI-RSD

Subject: Monthly Progress Report #9
Contract #DAAH01-69-C-0296 for period May 1-31, 1969
"Dynamics of Propellant in a Spinning Container and Description by a Mechanical Model."

Gentlemen:

During the subject period the translationally forced oscillation case has been solved for the spinning region I. Previous results were wrong, since the excitation was applied in the tank-fixed system. The acceleration potential has now been divided into two parts of which one represents a wave of frequency $(\Omega + \Omega_o)$ in positive angular direction, and where the other is a wave of frequency $(\Omega - \Omega_o)$ in negative angular direction. The equations were solved for an inertial coordinate system, in which the spinning container was excited by $u = U \cos(\Omega_o t + \theta) \cos \Omega t$. For both wave forms the elliptic and hyperbolic differential equations had to be solved, i.e. we have to distinguish the forcing-frequency ranges:

1) $\Omega > \Omega_o$ (as obtained from $2\Omega_o < \Omega + \Omega_o$) } elliptic case
2) $\Omega > 3\Omega_o$ (as obtained from $2\Omega_o < \Omega - \Omega_o$)

3) $\Omega < \Omega_o$ (as obtained from $2\Omega_o > \Omega + \Omega_o$) } Hyperbolic case
4) $\Omega < 3\Omega_o$ (as obtained from $2\Omega_o > \Omega - \Omega_o$)

In order to cover the forcing frequency range Ω we have to consider the first range $(0 \leq \Omega \leq \Omega_o)$, in which both waves are solutions of the hyperbolic case, then the frequency range $\Omega_o \leq \Omega < 3\Omega_o$, where the $(\Omega + \Omega_o)$ -wave is obtained from the solution of the elliptic case and the second wave $(\Omega - \Omega_o)$ from the hyperbolic case. The following frequency range $\Omega > 3\Omega_o$, but $\Omega^2/4g$ small, is obtained from the solution of both $(\Omega \pm \Omega_o)$ the elliptic differential equations.

The procedure of obtaining response curves is based on a lengthy computational process, since for each Ω and Ω_o new eigenvalues have to be determined from the determinant (59), where ω is substituted by $\Omega \pm \Omega_o$.

In a project meeting, held in Mr. Goessling's office on May 12, 1969 at the U.S. Army Missile Command, Redstone Arsenal, Alabama, the progress to that time has been reported and the difficulty encountered in the forced oscillation case had been explained. Further discussion about the eigenvalues and the natural frequencies took place with Mr. W. Neale and Mr. J. Sofferis.

Sincerely yours,

H. F. Bauer, Professor

HFB:jv

cc: AMSMI-RSD - 2
AMSMI-RSP - 1
AMSMI-IWD - 1

GEORGIA INSTITUTE OF TECHNOLOGY
School of Engineering Science and Mechanics

Monthly Cost and Performance Report # 9 Month of: May, 1969

Project Title: "Dynamics of Propellant in a Spinning Container and Description and a Mechanical Model"

Project No: B-903 Contract No: DAAH01-69-C-0296 Project Director: Dr. H. F. Bauer

Sponsor: U. S. Army Missile Command; Redstone Arsenal, Alabama

		FOR MONTH	TO DATE
Manhours Expended:	Project Director	115	536
	Graduate Assistants	144	984
Funds Expended:			
Personal Services		$1,566.49	$ 8,938.50
Material and Operating Expenses			
Travel		68.86	286.26
Computer Use Charges			
Overhead		892.90	5,106.30
Total Funds Expended		$2,528.25	$14,331.06
Percentage of Work Completed:		14%	69%

Project Director

ENGINEERING COLLEGE

GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL OF ENGINEERING SCIENCE
AND MECHANICS
225 NORTH AVENUE, N.W.
ATLANTA, GEORGIA 30332

June 30, 1969

U. S. Army Missile Command
Redstone Arsenal, Alabama 35809

Attention: AMSMI-RSD

Subject; Monthly Progress Report #10
Contract #DAAH01-69-C-0296 for period June 1-30, 1969
"Dynamics of Propellant in a Spinning Container and Description and a Mechanical Model."

Gentlemen:

During the subject period the pressure distribution, velocity distribution, liquid force, liquid movement as well as the free liquid surface elevation about the stationary spinning surface have been obtained analytically and evaluated. This was performed for the case of small spin-frequency Ω_o, i.e., small $\Omega_o^2/4g < 1$, for the elliptic and hyperbolic case. The velocity distribution inside the container was included in the analysis for reasons of possible baffling of the container if necessary for dynamic stability considerations. The baffle has its largest effect there where the velocity of the liquid exhibits large values.

The result of the analysis was presented in written form, where the acceleration potential for all the above-mentioned cases was given.

In a letter, computations for the sloshing frequency during the accelerating actual flight phase were requested, and the necessary data for the numerical work were presented. Other natural frequency computations were also requested to find the effect of liquid height and spin-frequency/acceleration upon the slosh frequencies. In a project meeting on June 25, 1969 at the U. S. Army Missile Command, Redstone Arsenal, Alabama, the progress to that time has been reported and the analytical expressions for the liquid-moment, pressure and velocity distribution, as well as the free liquid surface elevation have been presented for numerical evaluation.

Sincerely,

H. F. Bauer
Professor

HFB:mw

cc: AMSMI-RSD - 2
AMSMI-RSP - 1
AMSMI-IWD - 1

GEORGIA INSTITUTE OF TECHNOLOGY
School of Engineering Science and Mechanics

Monthly Cost and Performance Report # 10   Month of: June, 1969

Project Title: "Dynamics of Propellant in a Spinning Container and Description and a Mechanical Model"

Project No: B-903   Contract No: DAAH01-69-C-0296   Project Director: Dr. H. F. Bauer

Sponsor: U. S. Army Missile Command; Redstone Arsenal, Alabama

		FOR MONTH	TO DATE *
Manhours Expended:	Project Director	115	651
	Graduate Assistants	144	1128
Funds Expended:			
Personal Services		$3,629.47 **	$12,587.47
Material and Operating Expenses			0
Travel		97.05	383.31
Computer Use Charges			0
Overhead		2,068.80	7,174.85
Total Funds Expended		$5,795.32	$20,145.63
Percentage of Work Completed:		14%	83%

* and **: See Reverse Side.

Project Director

ENGINEERING COLLEGE

GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL OF ENGINEERING SCIENCE
AND MECHANICS

225 NORTH AVENUE, N.W.
ATLANTA, GEORGIA 30332

July 31, 1969

U.S. Army Missile Command
Redstone Arsenal, Alabama 35809

Attention: AMSMI-RSD

Subject: Monthly Progress Report #11
Contract #DAAH01-69-C-0296 for period July 1-31, 1969
"Dynamics of Propellant in a Spinning Container and
Description and a Mechanical Model."

Gentlemen:

During the subject period the forced oscillations of liquid for the case of very large spin-frequency Ω_o (i.e., when $\Omega_o^2/4g \gg 1$, such that the liquid surface is nearly vertical) has been investigated. It is found that this problem can not be treated by the same procedure as was used for the case of small values of spin-frequency Ω_o. Therefore, further investigation is being undertaken now.

The numerical results of sloshing frequencies ω_{mn} during the accelerating actual flight phase have been listed and plotted as shown in Table 1 and Figures 1,2. With the spinning frequency Ω_o as a parameter, some numerical results of sloshing frequencies ω_{mn} for different liquid heights have been listed and plotted, as shown in Tables 2-8 and Figures 3-11. Examination of Figures 1,2 shows that the values of sloshing frequencies ω_{mn} have strongly changed during the actual flight time interval from $t=2$ sec to $t=4$ sec. Particularly, the frequencies of frequencies of ω_{11}, ω_{12}, ω_{13} are increasing quite largely during that interval; however, ω_{21}, ω_{22}, ω_{23} are increasing sharply from $t=2$ sec until $t=3$ sec then decreasing again as shown in Figure 1. In Figures 3-11 we can see that the sloshing frequencies ω_{mn} are almost independent of the liquid height when the ratio of liquid height h (at the static equilibrium position) over the tank radius a (i.e., h/a) is larger than 1. It is of interest to note that for $h/a>1$, the sloshing frequencies ω_{01}, ω_{02}, ω_{03} (symmetric modes) are independent of the spinning velocities Ω_o (at least for $\Omega_o=0$ to $\Omega_o=3.0$).

I would like to have the results of the numerical work as requested in the last meeting on June 25, 1969, to compute liquid forces, and moments (as well as liquid elevations) for the case of forced oscillations of liquid under small values of Ω_o (both elliptic and hyperbolic cases) if they are available.

Sincerely yours,

Helmut F. Bauer, Professor

HFB:ps

cc: AMSMI-RSD - 2
AMSMI-RSP - 1
AMSMI-IWD - 1

GEORGIA INSTITUTE OF TECHNOLOGY
School of Engineering Science and Mechanics

Monthly Cost and Performance Report # 11 Month of: July

Project Title: "Dynamics of Propellant in a Spinning Container and Description and a Mechanical Model"

Project No: B-903 Contract No: DAAH01-69-C-0296 Project Director: Dr. H. F. Bauer

Sponsor: U. S. Army Missile Command; Redstone Arsenal, Alabama

		FOR MONTH	TO DATE
Manhours Expended:	Project Director	**115** *	651
	Graduate Assistants	115	1243
Funds Expended:			
Personal Services		$445.00	$13,032.47
Material and Operating Expenses			0
Travel			383.31
Computer Use Charges			0
Overhead		253.65	7,428.50
Total Funds Expended		$698.65	$20,844.28
Percentage of Work Completed:		2%	85%

* Project Director, Dr. Bauer, is not on project payroll during Summer Quarter but is keeping up with the work of the project on his own time.

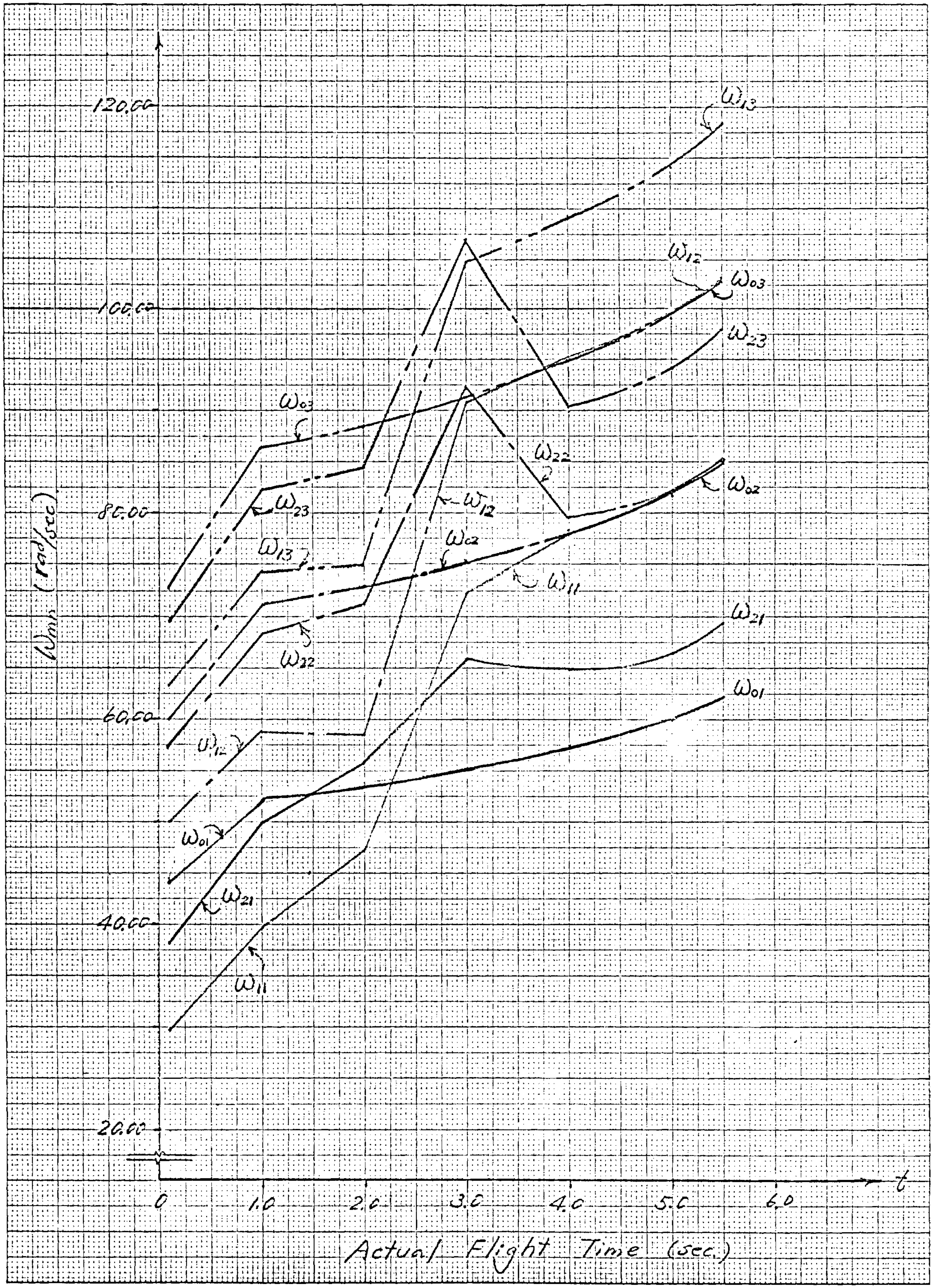
Milton E. Raville, Director
Engineering Science and Mechanics

Project Director

Actual Flight Time

t (sec) / ω_{mn} (rad/sec)	0.10	1.0	2.0	3.0	4.0	5.0	5.5
ω_{01}	43.9661	52.1588	53.5138	55.2256	57.3872	59.8302	62.1773
ω_{02}	60.0033	71.1844	73.0336	75.3699	78.3198	81.6541	84.8574
ω_{03}	72.6828	86.2267	88.4667	91.2966	94.8700	98.9088	102.7891
ω_{11}	29.6606	39.5046	47.1715	72.2582	78.2489	82.1050	85.2864
ω_{12}	50.0368	58.7030	58.4942	90.6627	95.1018	99.3352	103.2177
ω_{13}	63.2744	74.2216	74.8526	104.7090	109.2572	114.0057	118.4705
ω_{21}	38.6349	49.7920	55.8233	65.8646	64.7597	66.4958	69.1867
ω_{22}	57.1610	68.3543	71.3002	92.1883	79.4839	81.9077	85.1886
ω_{23}	69.4712	82.3790	84.4393	106.9186	90.3078	94.2400	97.9313
ω_{31}	45.3239	57.2916	62.6064	70.8058	70.7413	73.0152	75.9391
ω_{32}	62.5982	74.9362	78.0749	85.4952	85.7854	88.8958	92.4242
ω_{33}	74.4502	88.6071	91.4162	97.5252	99.0342	102.9620	107.0223
ω_{41}	50.9917	63.7251	68.7216	76.1746	76.7312	79.3793	82.5432
ω_{42}	67.3709	80.6132	83.8173	90.2562	91.5646	95.0428	98.8035
ω_{43}	78.7450	93.7463	96.7118	102.0799	104.5885	108.8085	113.0945
ω_{51}	56.0097	69.4661	74.2938	81.3368	82.3596	85.3197	88.7108
ω_{52}	71.7214	85.7752	89.0525	95.0959	96.9749	100.7442	104.7241
ω_{53}	82.7006	98.4534	101.5352	106.7158	109.6823	114.1475	118.6410

Table 1.



120.00
100.00
80.00
60.00
40.00
20.00
ω_{mn} (rad/sec)
0
1.0
2.0
3.0
4.0
5.0
6.0
t
Actual Flight Time (sec.)
ω_{13}
ω_{12}
ω_{03}
ω_{23}
ω_{22}
ω_{02}
ω_{11}
ω_{21}
ω_{01}

Fig. 1.

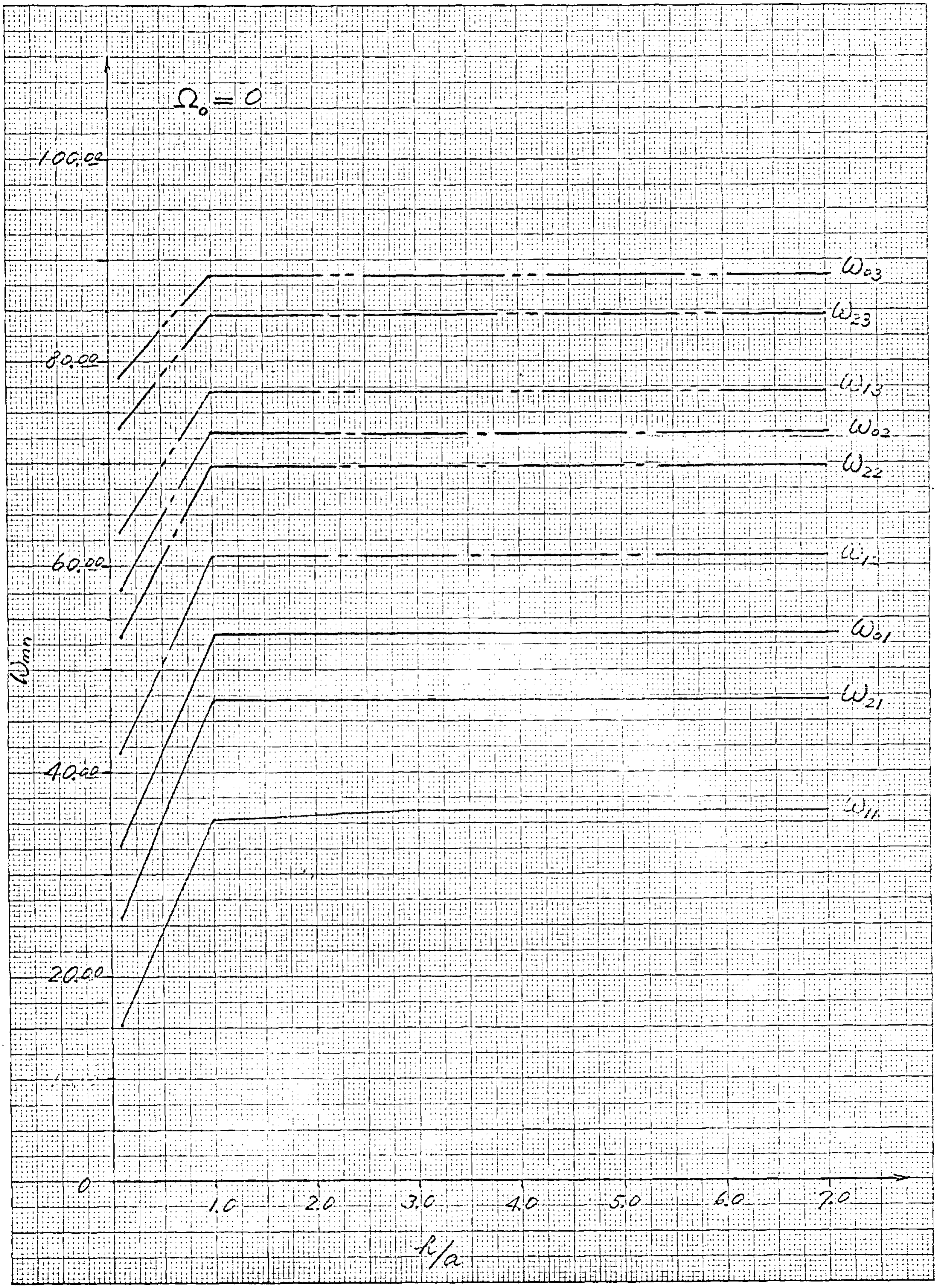

120.00
100.00
80.00
60.00
40.00
ω_{mn} (rad/sec)
ω_{53}
ω_{43}
ω_{33}
ω_{52}
ω_{42}
ω_{32}
ω_{51}
ω_{41}
ω_{31}
0
1.0
2.0
3.0
4.0
5.0
6.0
t
Actual Flight Time (sec)

Fig. 2

$\Omega_0 = 0$

ω_{mn} \ ℓ/a	0.1	1.0	2.0	3.0	4.0	5.0	6.0	7.0
ω_{01}	32.8113	53.4941	53.5138	53.5138	53.5138	53.5138	53.5138	53.5138
ω_{02}	57.8264	73.0336	73.0336	73.0336	73.0336	73.0336	73.0336	73.0336
ω_{03}	78.7970	88.4667	88.4667	88.4667	88.4667	88.4667	88.4667	88.4667
ω_{11}	15.2314	35.1268	36.0747	36.1010	36.1017	36.1018	36.1018	36.1018
ω_{12}	41.8204	60.9006	60.9028	60.9028	60.9028	60.9028	60.9028	60.9028
ω_{13}	63.2465	77.0151	77.0151	77.0151	77.0151	77.0151	77.0151	77.1513
ω_{21}	25.5895	46.9201	47.0247	47.0249	47.0249	47.0249	47.0249	47.0249
ω_{22}	53.1594	69.5741	69.5742	69.5742	69.5742	69.5742	69.5742	69.5742
ω_{23}	73.5255	84.5577	84.5577	84.5577	84.5577	84.5577	84.5577	84.5577
ω_{31}	34.7604	55.1541	55.1665	55.1666	55.1665	55.1665	55.1665	55.1665
ω_{32}	62.1236	76.1921	76.1921	76.1921	76.1921	76.1921	76.1921	76.1921
ω_{33}	81.6685	90.6179	90.6179	90.6179	90.6179	90.6179	90.6179	90.6179
ω_{41}	43.3000	62.0637	62.0652	62.0652	62.0652	62.0652	62.0652	62.0652
ω_{42}	70.0508	82.0012	82.0012	82.0012	82.0012	82.0012	82.0012	82.0012
ω_{43}	88.5357	95.8454	95.8454	95.8454	95.8454	95.8454	95.8454	95.8454
ω_{51}	51.2867	68.1726	68.1728	68.1728	68.1728	68.1728	68.1728	68.1728
ω_{52}	77.2255	87.2965	87.2965	87.2965	87.2965	87.2965	87.2965	87.2965
ω_{53}	94.6992	100.6599	100.6599	100.6599	100.6599	100.6599	100.6599	100.6599

Table 2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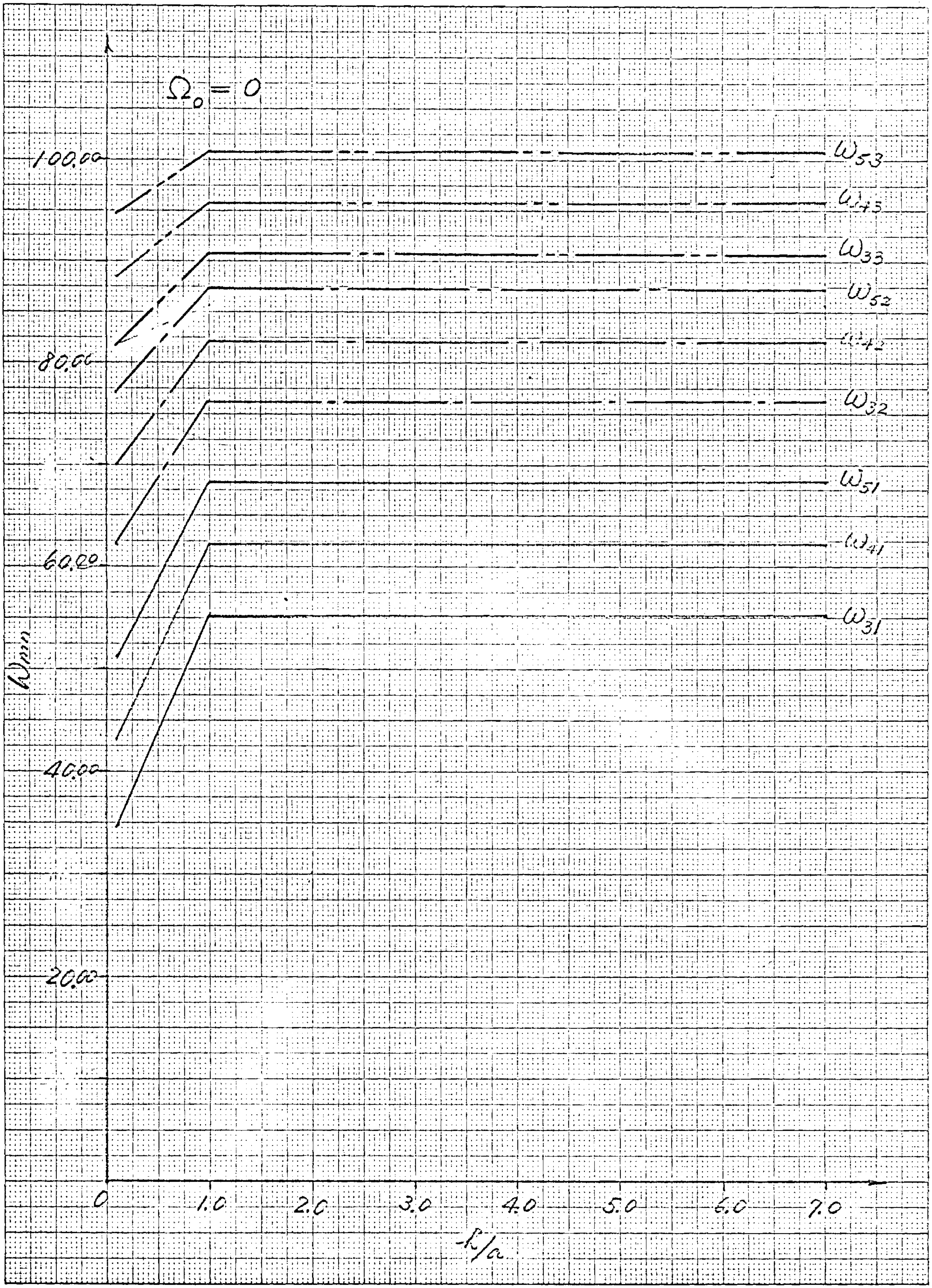
100.00
80.00
60.00
40.00
20.00
0
ω_{mn}
ω_{03}
ω_{23}
ω_{13}
ω_{02}
ω_{22}
ω_{12}
ω_{01}
ω_{21}
ω_{11}
1.0
2.0
3.0
4.0
5.0
6.0
7.0
h/a

Fig. 3.



$\Omega_0 = 0$
100.00
80.00
60.00
40.00
20.00
ω_{mn}
ω_{53}
ω_{43}
ω_{33}
ω_{52}
ω_{42}
ω_{32}
ω_{51}
ω_{41}
ω_{31}
0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
$-k/a$

Fig. 4

$\Omega_0 = 0.5$

h/a	0.1	1.0	2.0	3.0	4.0	5.0	6.0	7.0
ω_{01}	186.1878	53.4941	53.5138	53.5138	53.5138	53.5138	53.5138	53.5138
ω_{02}	328.1362	73.0336	73.0336	73.0336	73.0336	73.0336	73.0336	73.0336
ω_{03}	447.1338	88.4667	88.4667	88.4667	88.4667	88.4667	88.4667	88.4667
ω_{11}	334.0566	47.2975	47.3541	47.3542	47.3542	47.3542	47.3542	47.3542
ω_{12}	452.9025	58.4210	58.4263	58.4263	58.4263	58.4263	58.4263	58.4263
ω_{13}	548.5838	74.8197	74.8197	74.8197	74.8197	74.8197	74.8197	74.8197
ω_{21}	236.6625	55.9170	55.9251	55.9251	55.9251	55.9251	55.9251	55.9251
ω_{22}	326.0470	71.3366	71.3366	71.3366	71.3366	71.3366	71.3366	71.3366
ω_{23}	415.9098	84.4367	84.4367	84.4367	84.4367	84.4367	84.4367	84.4367
ω_{31}	271.8539	62.6830	62.6841	62.6841	62.6841	62.6841	62.6841	62.6841
ω_{32}	373.3548	78.1072	78.1072	78.1072	78.1072	78.1072	78.1072	78.1072
ω_{33}	470.8162	91.4304	91.4304	91.4304	91.4304	91.4304	91.4304	91.4304
ω_{41}	310.7147	68.7877	68.7878	68.7878	68.7878	68.7878	68.7878	68.7878
ω_{42}	415.5066	83.8457	83.8457	83.8457	83.8457	83.8457	83.8457	83.8457
ω_{43}	509.7331	96.7256	96.7256	96.7256	96.7256	96.7256	96.7256	96.7256
ω_{51}	349.0899	74.3529	74.3529	74.3529	74.3529	74.3529	74.3529	74.3529
ω_{52}	454.3111	89.0783	89.0783	89.0783	89.0783	89.0783	89.0783	89.0783
ω_{53}	544.3287	101.5484	101.5484	101.5484	101.5484	101.5484	101.5484	101.5484

Table 3

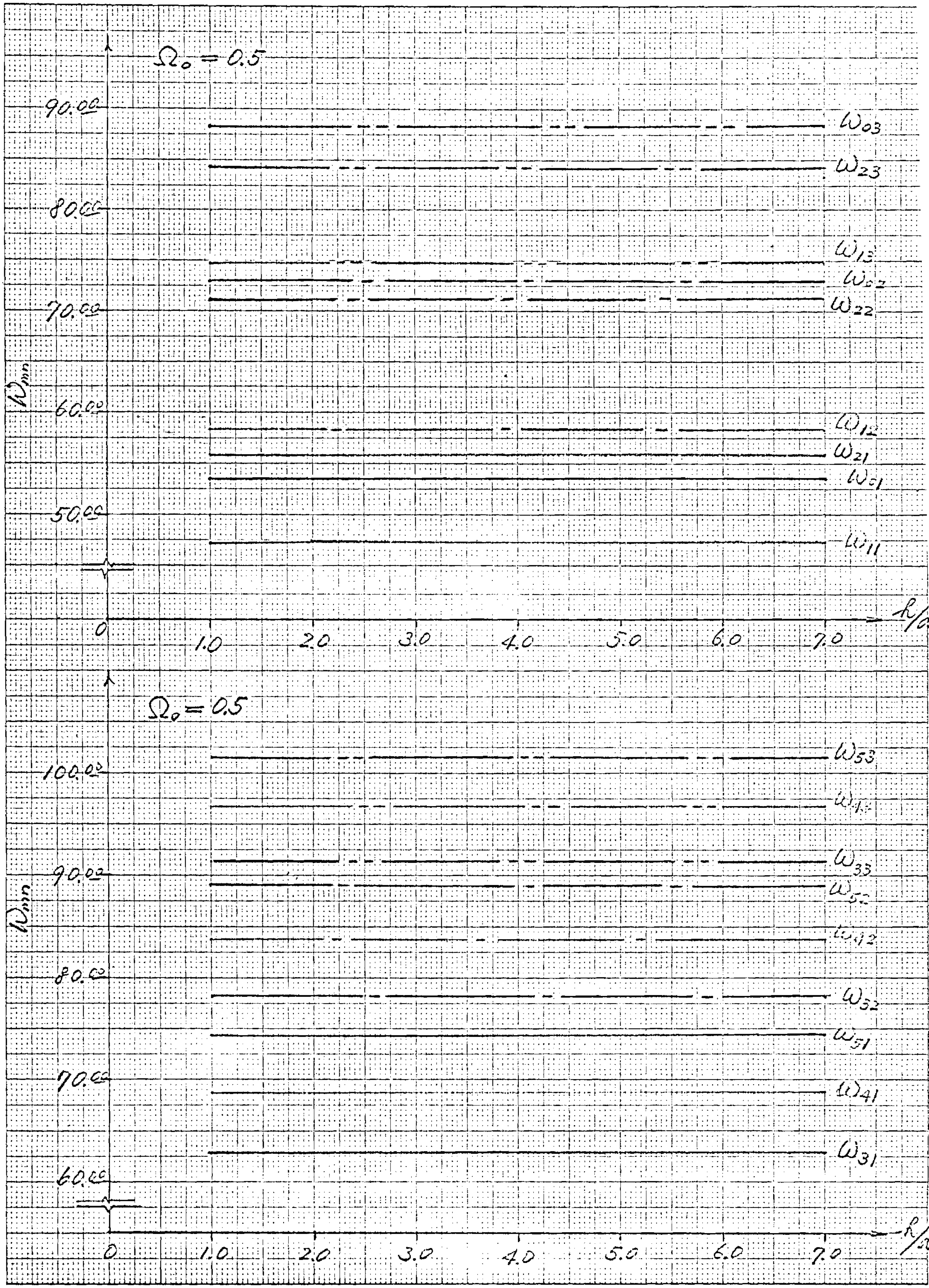
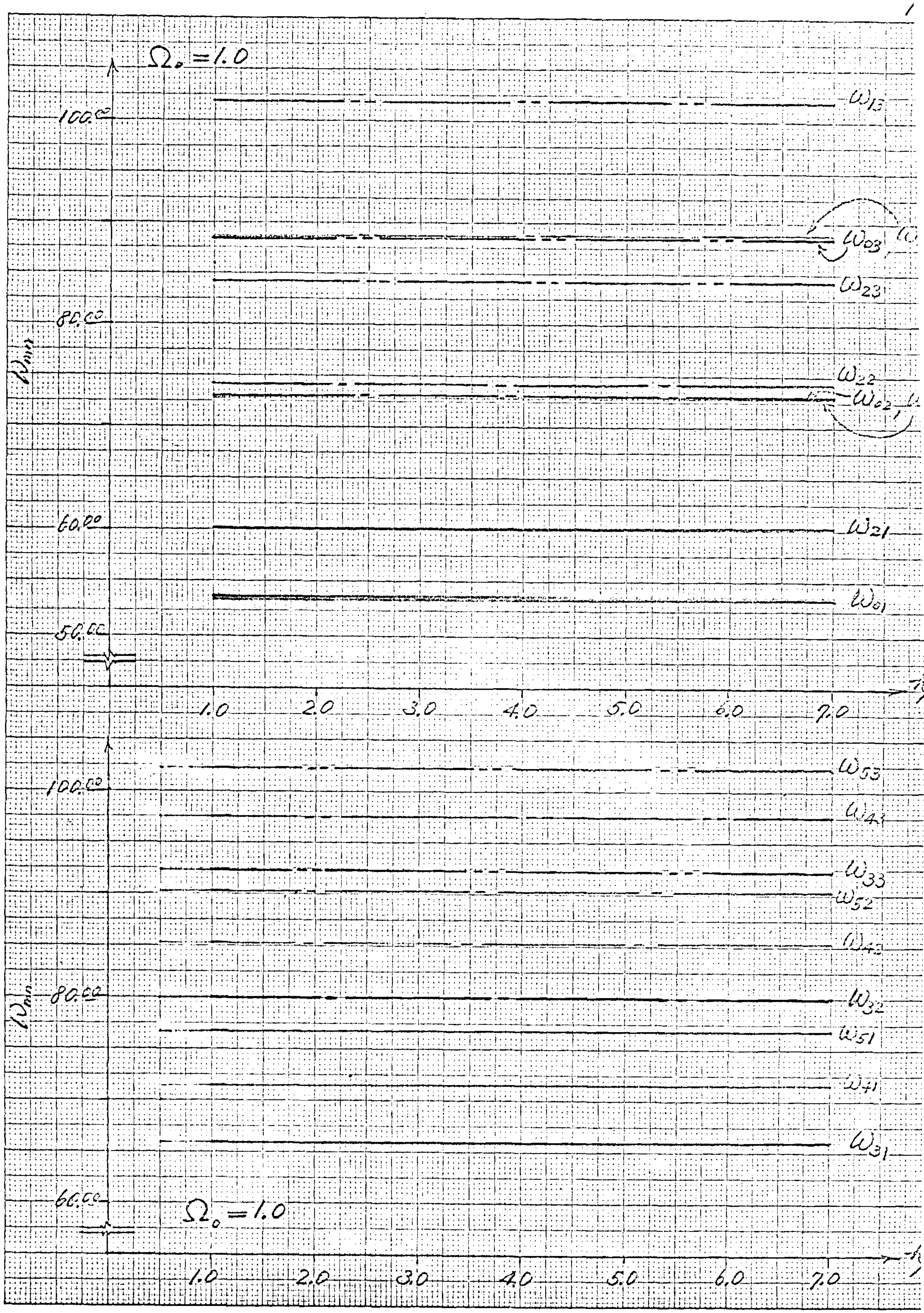

$\Omega_o = 0.5$
ω_{mn}
90.00
80.00
70.00
60.00
50.00
ω_{03}
ω_{23}
ω_{13}
ω_{02}
ω_{22}
ω_{12}
ω_{21}
ω_{01}
ω_{11}
0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
$\Omega_o = 0.5$
100.00
90.00
80.0
70.00
60.00
ω_{53}
ω_{33}
ω_{42}
ω_{32}
ω_{51}
ω_{41}
ω_{31}

Fig. 5

$\Omega_0 = 1.0$

ω_{mn} \ h/a	0.1	1.0	2.0	3.0	4.0	5.0	6.0	7.0
ω_{01}	1056.5232	53.4941	53.5138	53.5138	53.5138	53.5138	53.5138	53.5138
ω_{02}	1862.0100	73.0336	73.0336	73.0336	73.0336	73.0336	73.0336	73.0336
ω_{03}	2537.2629	88.4667	88.4667	88.4667	88.4667	88.4667	88.4667	88.4667
ω_{11}	1722.4620	72.9785	72.9785	72.9785	72.9785	72.9785	72.9785	72.9785
ω_{12}	2531.8874	88.6866	88.6866	88.6866	88.6866	88.6866	88.6866	88.6866
ω_{13}	3095.5540	101.8848	101.8848	101.8848	101.8848	101.8848	101.8848	101.8848
ω_{21}	1499.2264	60.3668	60.3688	60.3688	60.3688	60.3688	60.3688	60.3688
ω_{22}	2135.3135	74.0975	74.0976	74.0976	74.0976	74.0976	74.0976	74.0976
ω_{23}	2329.0439	84.2146	84.2146	84.2146	84.2146	84.2146	84.2146	84.2146
ω_{31}	1679.9365	65.9519	65.9522	65.9522	65.9522	65.9522	65.9522	65.9522
ω_{32}	2238.4197	79.9840	79.9840	79.9840	79.9840	79.9840	79.9840	79.9840
ω_{33}	2725.1638	92.3437	92.3437	92.3437	92.3437	92.3437	92.3437	92.3437
ω_{41}	1886.0727	71.5401	71.5401	71.5401	71.5401	71.5401	71.5401	71.5401
ω_{42}	2443.6903	85.3758	85.3758	85.3758	85.3758	85.3758	85.3758	85.3758
ω_{43}	2932.4253	97.5244	97.5244	97.5244	97.5244	97.5244	97.5244	97.5244
ω_{51}	2091.8830	76.7901	76.7901	76.7901	76.7901	76.7901	76.7901	76.7901
ω_{52}	2647.0976	90.4222	90.4222	90.4222	90.4222	90.4222	90.4222	90.4222
ω_{53}	3122.0888	102.2750	102.2750	102.2750	102.2750	102.2750	102.2750	102.2750

Table 4



$\Omega_o = 1.0$
100.00
80.00
60.00
50.00
ω_{nm}
ω_{13}
ω_{03}
ω_{23}
ω_{22}
ω_{02}
ω_{21}
ω_{01}
1.0
2.0
3.0
4.0
5.0
6.0
7.0
100.00
80.00
60.00
ω_{nm}
ω_{53}
ω_{43}
ω_{33}
ω_{52}
ω_{42}
ω_{32}
ω_{51}
ω_{41}
ω_{31}
$\Omega_o = 1.0$
1.0
2.0
3.0
4.0
5.0
6.0
7.0

Fig. 6

$\Omega_0 = 1.5$

k/a	0.1	1.0	2.0	3.0	4.0	5.0	6.0	7.0
ω_{01}	5995.2460	53.4941	53.5138	53.5138	53.5138	53.5138	53.5138	53.5138
ω_{02}	10565.9842	73.0336	73.0336	73.0336	73.0336	73.0336	73.0336	73.0336
ω_{03}	14397.7092	88.4667	88.4667	88.4667	88.4667	88.4667	88.4667	88.4667
ω_{11}	8861.1678	70.7895	70.7896	70.7896	70.7896	70.7896	70.7896	70.7896
ω_{12}	141770702	88.0467	88.0467	88.0467	88.0467	88.0467	88.0467	88.0467
ω_{13}	17485.2111	101.5594	101.5594	101.5594	101.5594	101.5594	101.5594	101.5594
ω_{21}	8833.1360	63.2032	63.2040	63.2040	63.2040	63.2040	63.2040	63.2040
ω_{22}	13783.3875	91.2055	91.2055	91.2055	91.2055	91.2055	91.2055	91.2055
ω_{23}	17976.5429	103.9622	103.9622	103.9622	103.9622	103.9622	103.9622	103.9622
ω_{31}	9923.0623	68.0913	68.0914	68.0914	68.0914	68.0914	68.0914	68.0914
ω_{32}	13337.0155	82.1437	82.1437	82.1437	82.1437	82.1437	82.1437	82.1437
ω_{33}	15978.1855	93.8305	93.8305	93.8305	93.8305	93.8305	93.8305	93.8305
ω_{41}	11091.9695	73.3582	73.3583	73.3583	73.3583	73.3583	73.3583	73.3583
ω_{42}	14320.0995	86.9581	86.9581	86.9581	86.9581	86.9581	86.9581	86.9581
ω_{43}	16928.2635	98.5347	98.5347	98.5347	98.5347	98.5347	98.5347	98.5347
ω_{51}	12241.7428	78.4054	78.4054	78.4054	78.4054	78.4054	78.4054	78.4054
ω_{52}	15380.5893	91.7383	91.7383	91.7383	91.7383	91.7383	91.7383	91.7383
ω_{53}	17933.4898	103.1105	103.1105	103.1105	103.1105	103.1105	103.1105	103.1105

Table 5

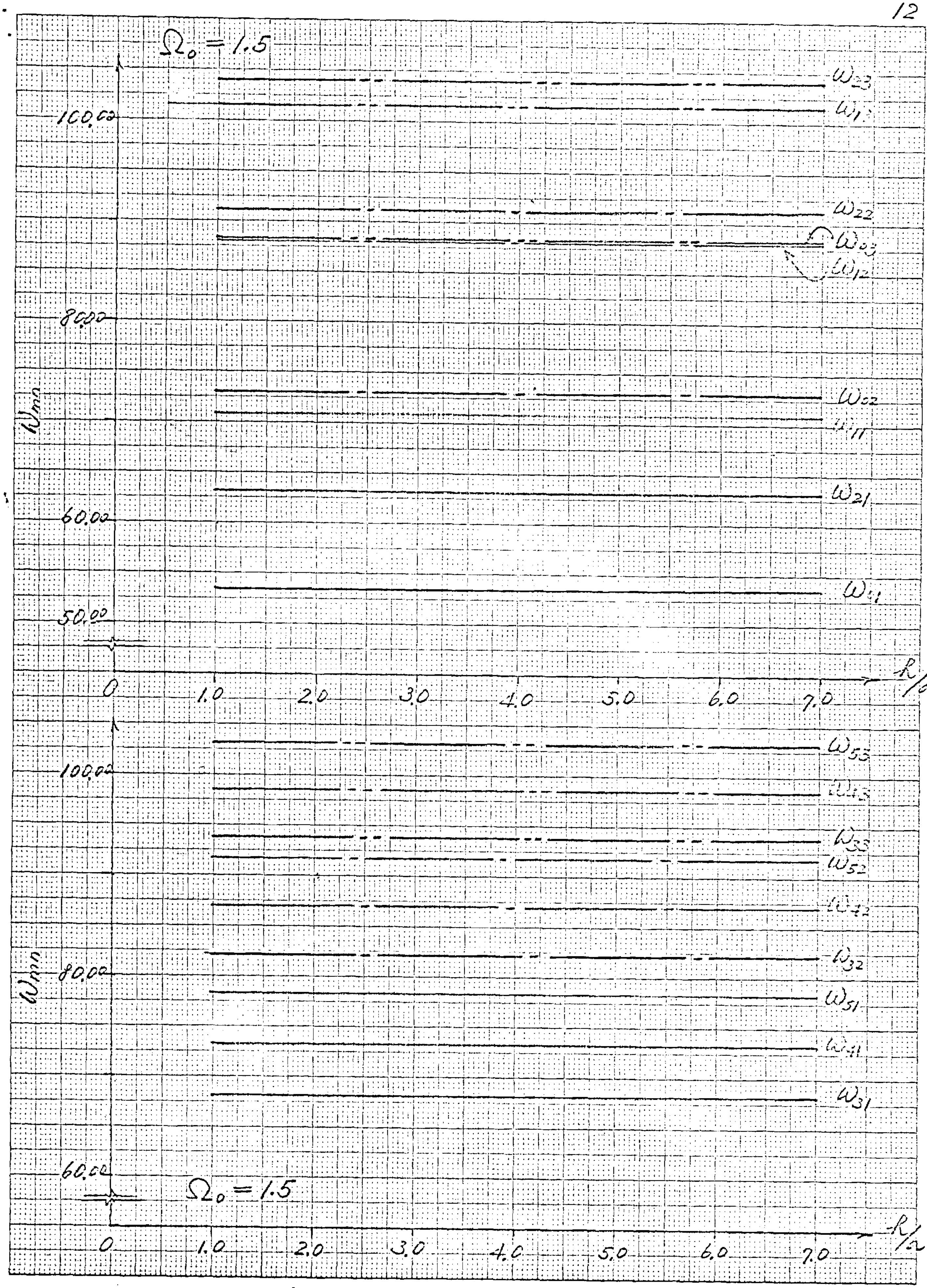
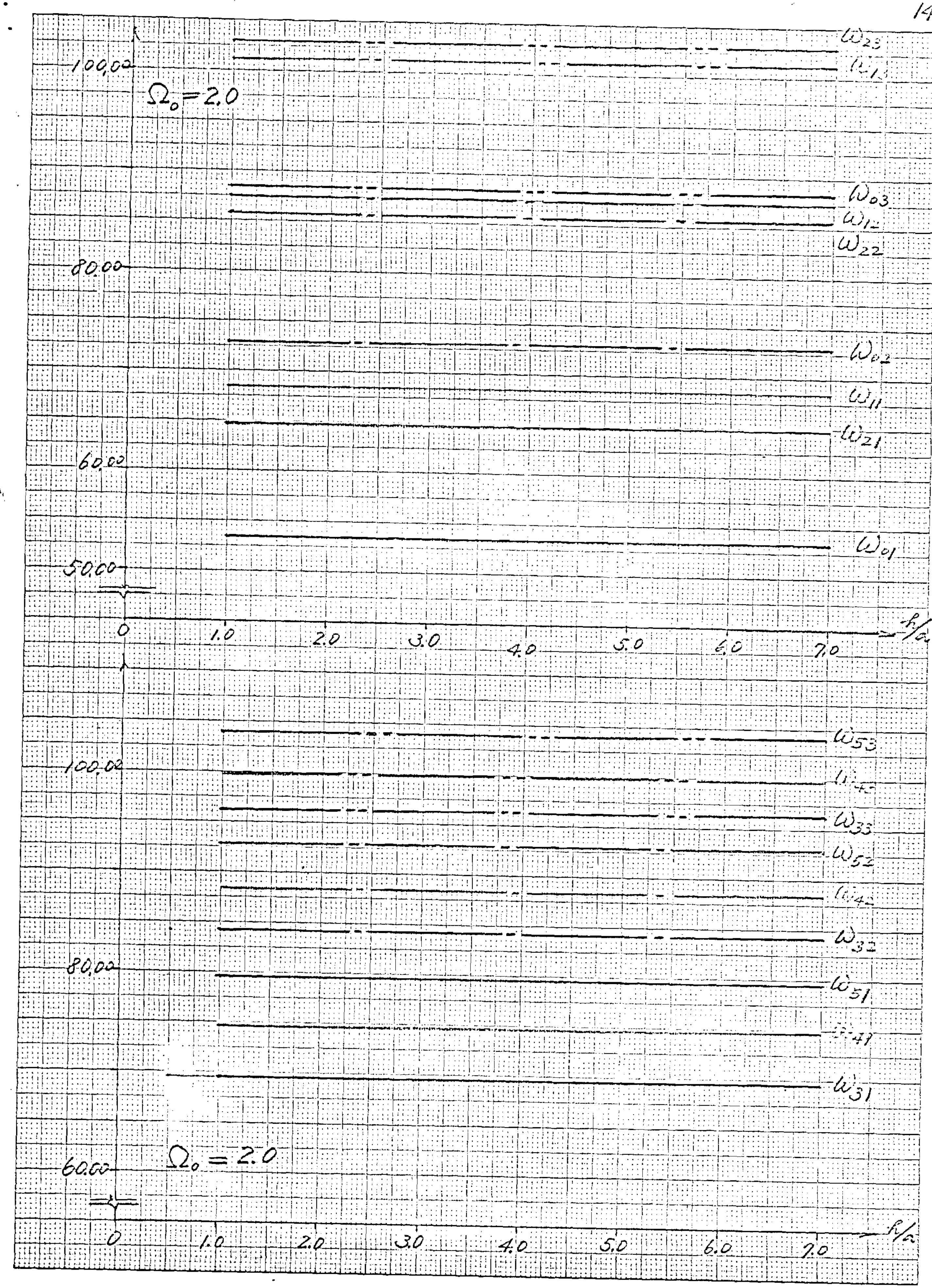

$\Omega_0 = 1.5$
ω_{23}
ω_{13}
100.00
ω_{22}
ω_{03}
ω_{12}
80.00
ω_{02}
ω_{11}
ω_{21}
60.00
ω_{01}
50.00
W_{mn}
0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
R/a
ω_{53}
ω_{43}
ω_{33}
ω_{52}
ω_{42}
ω_{32}
ω_{51}
ω_{41}
ω_{31}
60.00
$\Omega_0 = 1.5$

Fig. 8

$\Omega_0 = 2.0$

ω_{mn} \ f/a	0.1	1.0	2.0	3.0	4.0	5.0	6.0	7.0
ω_{01}	34020.0497	53.4941	53.5138	53.5138	53.5138	53.5138	53.5138	53.5138
ω_{02}	59956.7235	73.0336	73.0336	73.0336	73.0336	73.0336	73.0336	73.0336
ω_{03}	81699.8639	88.4667	88.4667	88.4667	88.4667	88.4667	88.4667	88.4667
ω_{11}	48520.1609	68.4701	68.4703	68.4703	68.4703	68.4703	68.4703	68.4703
ω_{12}	79475.2902	87.4675	87.4675	87.4675	87.4675	87.4675	87.4675	87.4675
ω_{13}	98820.1731	101.2754	101.2754	101.2754	101.2754	101.2754	101.2754	101.2754
ω_{21}	50951.2050	64.7663	64.7668	64.7668	64.7668	64.7668	64.7668	64.7668
ω_{22}	76936.6930	85.7648	85.7648	85.7648	85.7648	85.7648	85.7648	85.7648
ω_{23}	100507.79	102.8930	102.8930	102.8930	102.8930	102.8930	102.8930	102.8930
ω_{31}	57525.4054	69.4824	69.4825	69.4825	69.4825	69.4825	69.4825	69.4825
ω_{32}	78175.2255	84.1849	84.1849	84.1849	84.1849	84.1849	84.1849	84.1849
ω_{33}	94616.6659	96.2225	96.2247	96.2225	96.2225	96.2225	96.2225	96.2225
ω_{41}	64278.4498	74.5989	74.5989	74.5989	74.5989	74.5989	74.5989	74.5989
ω_{42}	83323.5322	88.4356	88.4356	88.4356	88.4356	88.4356	88.4356	88.4356
ω_{43}	97962.0894	99.7772	99.7772	99.7772	99.7772	99.7772	99.7772	99.7772
ω_{51}	70814.1260	79.5334	79.5334	79.5334	79.5334	79.5334	79.5334	79.5334
ω_{52}	88993.1698	92.9509	92.9509	92.9509	92.9509	92.9509	92.9509	92.9509
ω_{53}	103121.63	104.0580	104.0580	104.0580	104.0580	104.0580	104.0580	104.0580

Table 6



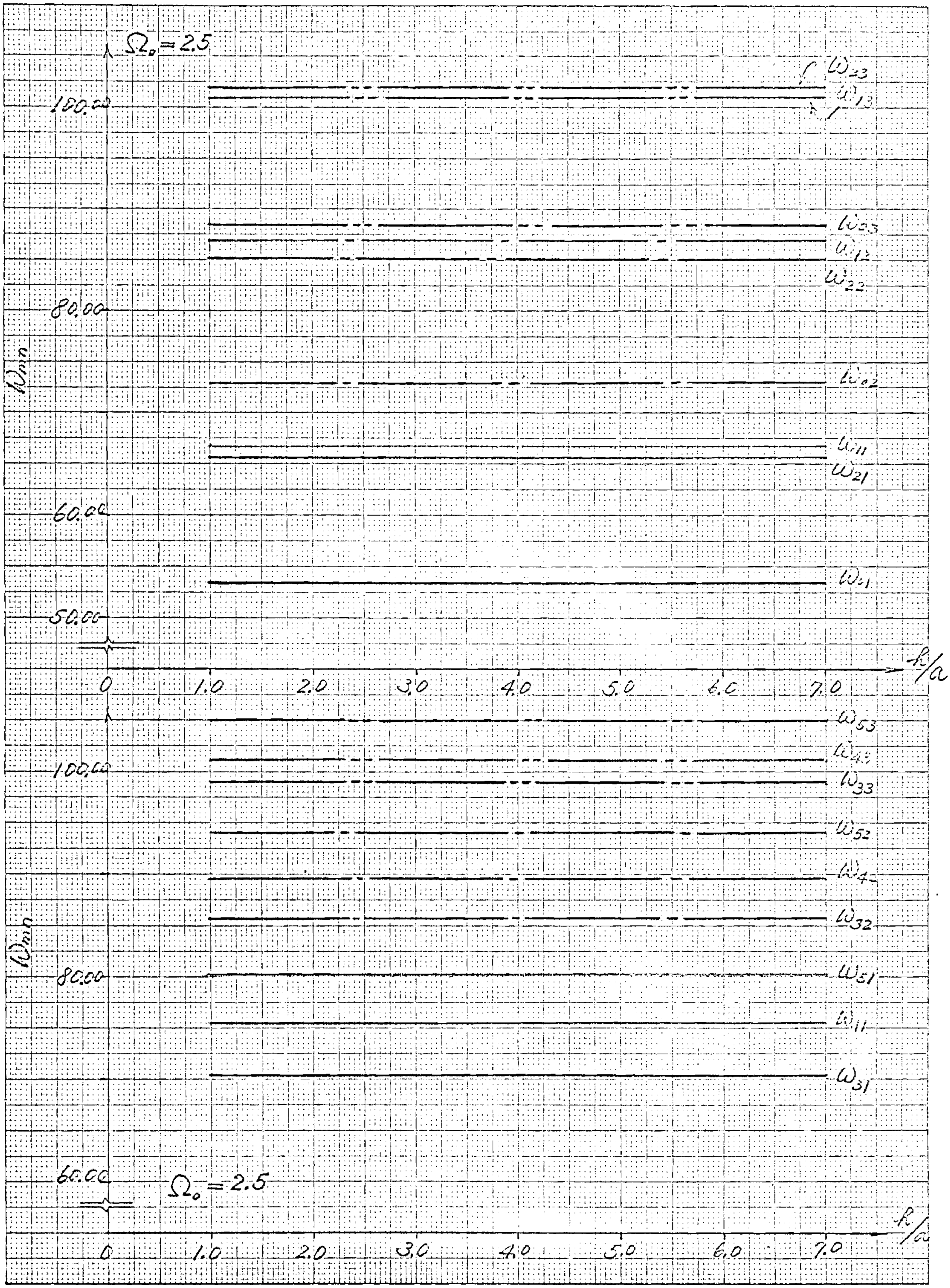
10 Millimeters to the Centimeter

Fig. 9

$\Omega_0 = 2.5$

k/a	0.1	1.0	2.0	3.0	4.0	5.0	6.0	7.0
ω_{01}	193046.92	53.4941	53.5138	53.5138	53.5138	53.5138	53.5138	53.5138
ω_{02}	340224.69	73.0336	73.0336	73.0336	73.0336	73.0336	73.0336	73.0336
ω_{03}	463606.24	88.4667	88.4667	88.4667	88.4667	88.4667	88.4667	88.4667
ω_{11}	271011.25	66.7236	66.7239	66.7239	66.7239	66.7239	66.7239	66.7239
ω_{12}	446054.63	86.9391	86.9391	86.9391	86.9391	86.9391	86.9391	86.9391
ω_{13}	558713.80	101.0201	101.0201	101.0201	101.0201	101.0201	101.0201	101.0201
ω_{21}	291714.74	65.6226	65.6230	65.6230	65.6230	65.6230	65.6230	65.6230
ω_{22}	43538838	85.1140	85.1140	85.1140	85.1140	85.1140	85.1140	85.1140
ω_{23}	563527.53	101.9780	101.9780	101.9780	101.9780	101.9780	101.9780	101.9780
ω_{31}	330639.73	70.3957	70.3958	70.3958	70.3958	70.3958	70.3958	70.3958
ω_{32}	451873.12	85.7094	85.7094	85.7094	85.7094	85.7094	85.7094	85.7094
ω_{33}	552711.91	98.9239	98.9239	98.9239	98.9239	98.9239	98.9239	98.9239
ω_{41}	369677.01	75.4635	75.4635	75.4635	75.4635	75.4635	75.4635	75.4635
ω_{42}	481266.09	89.6631	89.6631	89.6631	89.6631	89.6631	89.6631	89.6631
ω_{43}	566391.50	101.1315	101.1315	101.1315	101.1315	101.1315	101.1315	101.1315
ω_{51}	407025.96	80.3437	80.3437	80.3437	80.3437	80.3437	80.3437	80.3437
ω_{52}	512605.15	93.9877	93.9877	93.9877	93.9877	93.9877	93.9877	93.9877
ω_{53}	592870.60	105.0590	105.0590	105.0590	105.0590	105.0590	105.0590	105.0590

Table 7



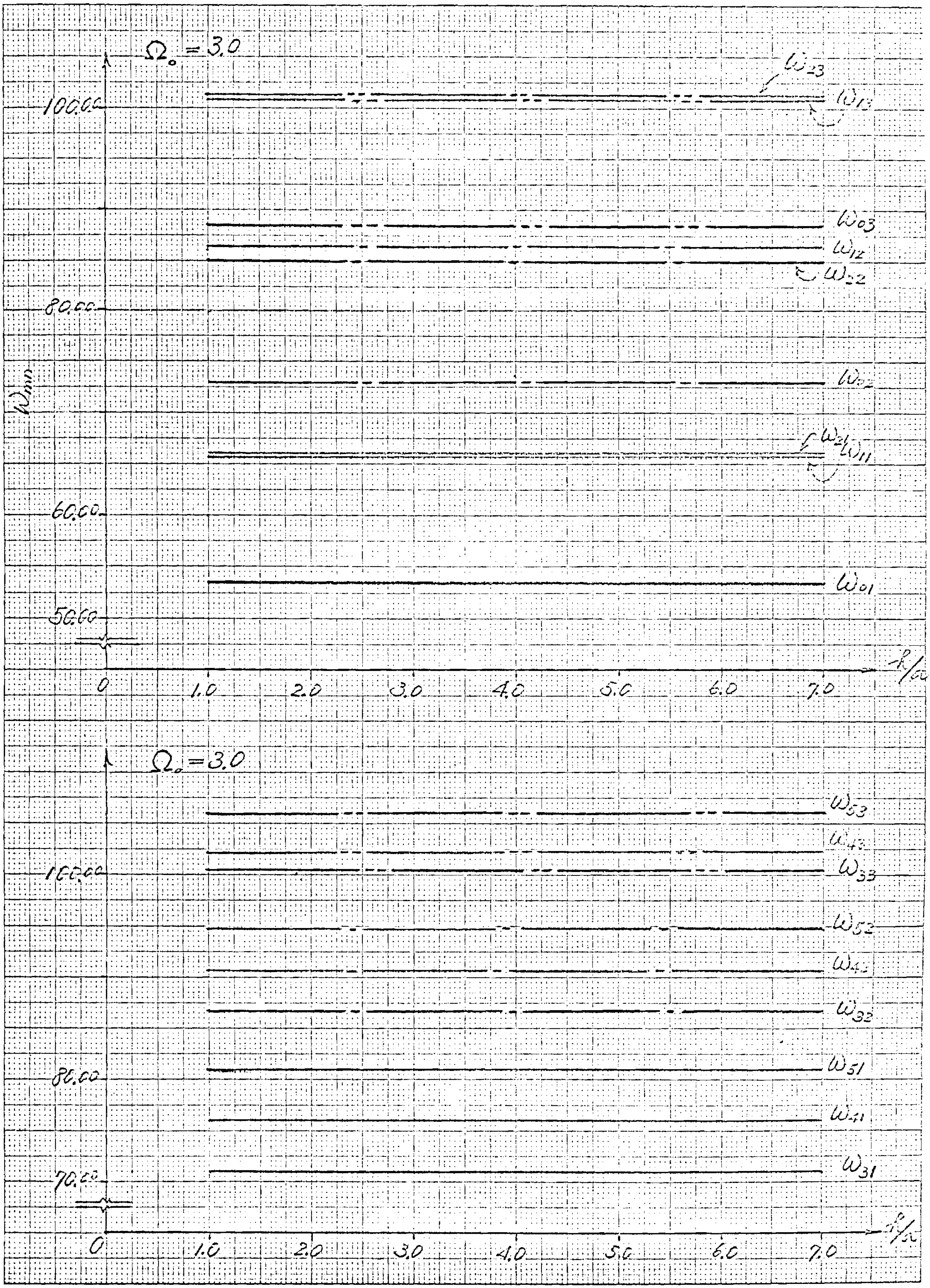
Millimeters to the Centimeter

Fig. 10

$\Omega_0 = 3.0$

h/a \ ω_{mn}	0.1	1.0	2.0	3.0	4.0	5.0	6.0	7.0
ω_{01}	1095445.6	53.4941	53.5138	53.5138	53.5138	53.5138	53.5138	53.5138
ω_{02}	1930606.5	73.0336	73.0336	73.0336	73.0336	73.0336	73.0336	73.0336
ω_{03}	2630735.6	88.4667	88.4667	88.4667	88.4667	88.4667	88.4667	88.4667
ω_{11}	1524107.1	65.6829	65.6834	65.6834	65.6834	65.6834	65.6834	65.6834
ω_{12}	2506562.6	86.4590	86.4590	86.4590	86.4590	86.4590	86.4590	86.4590
ω_{13}	3159843.7	100.7867	100.7867	100.7867	100.7867	100.7867	100.7867	100.7867
ω_{21}	1664650.1	66.1337	66.1341	66.1341	66.1341	66.1341	66.1341	66.1341
ω_{22}	2467763.4	84.9557	84.9557	84.9557	84.9557	84.9557	84.9557	84.9557
ω_{23}	3170050.3	101.2722	101.2722	101.2722	101.2722	101.2722	101.2722	101.2722
ω_{31}	1892128.8	71.0166	71.0167	71.0167	71.0167	71.0167	71.0167	71.0167
ω_{32}	2591533.8	86.6924	86.6924	86.6924	86.6924	86.6924	86.6924	86.6924
ω_{33}	3173014.8	100.4105	100.4105	100.4105	100.4105	100.4105	100.4105	100.4105
ω_{41}	2117087.5	76.0833	76.0833	76.0833	76.0833	76.0833	76.0833	76.0833
ω_{42}	2764075.1	90.5989	90.5989	90.5989	90.5989	90.5989	90.5989	90.5989
ω_{43}	3261839.9	102.3698	102.3698	102.3698	102.3698	102.3698	102.3698	102.3698
ω_{51}	2330687.6	80.9421	80.9421	80.9421	80.9421	80.9421	80.9421	80.9421
ω_{52}	2941250.8	94.8246	94.8246	94.8246	94.8246	94.8246	94.8246	94.8246
ω_{53}	3403421.4	106.0181	106.0181	106.0181	106.0181	106.0181	106.0181	106.0181

Table 8



10 Millimeters to the Centimeter

Fig. 11.

GEORGIA INSTITUTE OF TECHNOLOGY

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AND MECHANICS

225 NORTH AVENUE, N.W.
ATLANTA, GEORGIA 30332

September 2, 1969

U. S. Army Missile Command
Redstone Arsenal, Alabama 35809

Attention: AMSMI-RSD

Subject: Monthly Progress Report #12
Contract #DAAH01 for period August 1-31, 1969
"Dynamics of Propellant in a Spinning Container and Description by a Mechanical Model."

Gentlemen:

During the subject period, for the case of small spinning frequency Ω_o, i.e., $\Omega_o^2/4g \ll 1$, the response of the liquid in the container to forced pitching excitation about the y-axis of the inertial system was determined. According to the traveling waves in the same direction as the rotation of the system and those in opposite direction of the rotation, the acceleration potential was split up into two parts, for which the solutions for the elliptic and hyperbolic cases were determined. Pressure distribution and velocity distribution in the liquid, as well as liquid force, moment and free surface elevation, were determined. This concludes the theoretical investigation of the liquid behavior for small $\Omega_o^2/4g$, which is applicable to the first flight phase. The forced oscillations for the case of very large spin-frequency Ω_o (i.e., when $\Omega_o^2/4g \gg 1$, such that the liquid surface is nearly vertical) were determined in the sense of two-dimensional problem. If the force of gravity is neglected, the two dimensional waves might constitute a good approximate description of the liquid motions for this case.

The numerical results which were enclosed in a letter from Mr. John Sofferis (refer to AMSMI-KOCS) have been checked roughly by using mathematical tables and calculator. It was found that these numerical results (one by computer, the other one by calculator) were in a good agreement. For the free oscillation case, as we can see from equations (69) and (70), the values of F_x and M_y are dominated by the hyperbolic functions of sine and cosine, so the coefficients of F_x and M_y as requested will not converge. However, from physical point of view, we know that F_x and M_y should have a finite value. For convenience of evaluation of such coefficients of F_x and M_y a modification of equations (69) and (71) has been made in such a way that the series in equations (69) and (71) are divided by cosh $(\xi_{in} h/a)$ for each n. This is permissible since A_{in} and B_{in} in equations (69) and (71) are unknown constants which will be determined by initial conditions. Similarly, in equations (80) and (81) the same modification will be made. Please refer to attached copies.

Sincerely yours,

Helmut F. Bauer, Professor

HFB:ps

cc: AMSMI-RSD - 2
AMSMI-RSP - 1
AMSMI-IWD - 1

$$F_x = \frac{m}{(1-k^2)} \sum_{n=1}^{\infty} (A_{1n}\cos\omega_{1n}t + B_{1n}\sin\omega_{1n}t)\Big\{\frac{\omega_{1n}}{h}\,[Y_1'(\xi_{1n})J_1(\xi_{1n})$$

$$- J_1'(\xi_{1n})Y_1(\xi_{1n})] \cdot \sinh\left(\frac{\xi_{1n}}{a}\,\bar{z}_a\right) - \frac{k}{h}\bigg(\Big[2\,\frac{\Omega_o}{\xi_{1n}}\,Y_1(\xi_{1n})$$

$$+ \omega_{1n}Y_1'(\xi_{1n})\Big]J_1(k\xi_{1n}) - \Big[2\,\frac{\Omega_o}{\xi_{1n}}\,J_1(\xi_{1n})$$

$$+ \omega_{1n}J_1'(\xi_{1n})\Big]Y_1(k\xi_{1n})\bigg)\sinh\left(\xi_{1n}\,\frac{\bar{z}_b}{a}\right)\Big\}\frac{1}{\cosh\left(\xi_{1n}\frac{h}{a}\right)} \qquad (69)$$

~~where~~

$$M_y = \frac{m}{(1-k^2)} \sum_{n=1}^{\infty} (A_{1n}\cos\omega_{1n}t + B_{1n}\sin\omega_{1n}t)\frac{1}{\cosh\left(\xi_{1n}\frac{h}{a}\right)}\Big\{\omega_{1n}[Y_1'(\xi_{1n})J_1(\xi_{1n})$$

$$- J_1'(\xi_{1n})Y_1'(\xi_{1n})]\bigg[\left(\frac{a}{\xi_{1n}}\right)\left(\frac{1}{h}\right)\left(1 - \cosh\left(\xi_{1n}\,\frac{\bar{z}_a}{a}\right)\right)$$

$$+ \left(\frac{\bar{z}_a}{h} - \frac{1}{2}\right)\sinh\left(\xi_{1n}\,\frac{\bar{z}_a}{a}\right)\bigg] - (k)\bigg(\Big[2\,\frac{\Omega_o}{\xi_{1n}}\,Y_1(\xi_{1n}) + \omega_{1n}Y_1'(\xi_{1n})\Big]J_1(\xi_{1n}k)$$

$$- \Big[2\,\frac{\Omega_o}{\xi_{1n}}\,J_1(\xi_{1n}) + \omega_{1n}J_1'(\xi_{1n})\Big]Y_1(\xi_{1n}k)\bigg)\cdot\bigg[\left(\frac{a}{\xi_{1n}}\right)\frac{1}{h}\left(1 - \cosh\left(\xi_{1n}\,\frac{\bar{z}_b}{a}\right)\right)$$

$$+ \left(\frac{\bar{z}_b}{h} - \frac{1}{2}\right)\sinh\left(\xi_{1n}\,\frac{\bar{z}_b}{a}\right)\bigg] + \left(\frac{a}{h}\right)\bigg(\Big[2\,\frac{\Omega_o}{\xi_{1n}}\,Y_1(\xi_{1n}) + \omega_{1n}Y_1'(\xi_{1n})\Big][J_2(\xi_{1n})$$

$$- k^2J_2(\xi_{1n}k)] - \Big[2\,\frac{\Omega_o}{\xi_{1n}}\,J_1(\xi_{1n}) + \omega_{1n}J_1'(\xi_{1n})\Big][Y_2(\xi_{1n}) - k^2Y_2(\xi_{1n})]\bigg)\Big\}, \qquad (71)$$

$$
\begin{aligned}
F_x = \frac{m}{(1-k^2)} \sum_{n=1}^{\infty} & [A_{1n}\cos\bar{\omega}_{1n}t + B_{1n}\sin\bar{\omega}_{1n}t]\left\{\frac{\bar{\omega}_{1n}}{h}[K_1'(\bar{\xi}_{1n})I_1(\bar{\xi}_{1n})\right. \\
& - I_1'(\bar{\xi}_{1n})K_1(\bar{\xi}_{1n})]\sinh\left(\bar{\xi}_{1n}\frac{\bar{Z}_a}{a}\right) - \left(\frac{k}{h}\right)\left[\left(2\frac{\Omega_o}{\bar{\xi}_{1n}}K_1(\bar{\xi}_{1n})\right.\right. \\
& \left. + \bar{\omega}_{1n}K_1'(\xi_{1n})\right)I_1(\bar{\xi}_{1n}k) - \left(2\frac{\Omega_o}{\bar{\xi}_{1n}}I_1(\bar{\xi}_{1n})\right. \\
& \left.\left.\left. + \bar{\omega}_{1n}I_1'(\bar{\xi}_{1n})\right)K_1(\bar{\xi}_{1n}k)\right]\sinh\left(\bar{\xi}_{1n}\frac{\bar{z}_b}{a}\right)\right\}\frac{1}{\cosh\left(\bar{\xi}_{1n}\frac{h}{a}\right)} \qquad (80)
\end{aligned}
$$

~~and yields the expression~~

$$
\begin{aligned}
M_y = \frac{m}{(1-k^2)} \sum_{n=1}^{\infty} & [A_{1n}\cos\bar{\omega}_{1n}t + B_{1n}\sin\bar{\omega}_{1n}t]\Big\{\bar{\omega}_{1n}[K_1'(\bar{\xi}_{1n})I_1(\bar{\xi}_{1n}) \\
& - I_1'(\bar{\xi}_{1n})K_1(\bar{\xi}_{1n})]\left[\left(\frac{a}{\bar{\xi}_{1n}}\right)\left(\frac{1}{h}\right)\left(1 - \cosh\left(\bar{\xi}_{1n}\frac{\bar{Z}_a}{a}\right)\right)\right. \\
& \left. + \left(\frac{\bar{Z}_a}{h} - \frac{1}{2}\right)\sinh\left(\bar{\xi}_{1n}\frac{\bar{Z}_a}{a}\right)\right] - (k)\left(\left[2\frac{\Omega_o}{\bar{\xi}_{1n}}K_1(\bar{\xi}_{1n}) + \bar{\omega}_{1n}K_1'(\bar{\xi}_{1n})\right]I_1(\bar{\xi}_{1n}k)\right. \\
& \left. - \left[2\frac{\Omega_o}{\bar{\xi}_{1n}}I_1(\bar{\xi}_{1n}) + \bar{\omega}_{1n}I_1'(\bar{\xi}_{1n})\right]K_1(\bar{\xi}_{1n}k)\right) \\
& \cdot \left[\left(\frac{a}{\bar{\xi}_{1n}}\right)\left(\frac{1}{h}\right)\left(1 - \cosh\left(\bar{\xi}_{1n}\frac{b}{a}\right)\right) + \left(\frac{\bar{Z}_b}{h} - \frac{1}{2}\right)\sinh\left(\bar{\xi}_{1n}\frac{\bar{Z}_b}{a}\right)\right] \\
& + \left(\frac{a}{h}\right)\left(\left[2\frac{\Omega_o}{\bar{\xi}_{1n}}K_1(\bar{\xi}_{1n}) + \bar{\omega}_{1n}K_1'(\xi_{1n})\right][I_2(\bar{\xi}_{1n}) - k^2I_2(\bar{\xi}_{1n}k)]\right. \\
& \left. + \left[2\frac{\Omega_o}{\bar{\xi}_{1n}}I_1(\bar{\xi}_{1n}) + \bar{\omega}_{1n}I_1'(\bar{\xi}_{1n})\right][K_2(\bar{\xi}_{1n}) - k^2K_2(\bar{\xi}_{1n}k)]\right)\Big\} \; \cancel{(81)}\; \frac{1}{\cosh\left(\bar{\xi}_{1n}\frac{h}{a}\right)}
\end{aligned}
$$

GEORGIA INSTITUTE OF TECHNOLOGY
School of Engineering Science and Mechanics

Monthly Cost and Performance Report # 12 Month of: August, 1969

Project Title: "Dynamics of Propellant in a Spinning Container and Description and a Mechanical Model"

Project No: B-903 Contract No: DAAH01-69-C-0296 Project Director: Dr. H. F. Bauer

Sponsor: U. S. Army Missile Command; Redstone Arsenal, Alabama

		FOR MONTH	TO DATE
Manhours Expended:	Project Director	115 *	766
	Graduate Assistants	115	1,358
Funds Expended:			
Personal Services		$445.00	$13,477.47
Material and Operating Expenses			0
Travel		49.87	433.18
Computer Use Charges			
Overhead		253.65	7,682.15
Total Funds Expended		$748.52	$21,592.80
Percentage of Work Completed:		2%	87%

* Project Director, Dr. Bauer, is not on project payroll during summer quarter, but is keeping up with the work of the project on his own time.

Project Director

ENGINEERING COLLEGE

GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL OF ENGINEERING SCIENCE
AND MECHANICS

225 NORTH AVENUE, N.W.
ATLANTA, GEORGIA 30332

October 8, 1969

U.S. Army Missile Command
Redstone Arsenal, Alabama 35809

Attention: AMSMI-RSD

Subject: Monthly Progress Report #13
Contract #DAAH01 for period September 1-30, 1969
"Dynamics of Propellant in a Spinning Container and Description by a Mechanical Model."

Gentlemen:

During the past period the mechanical model for the description of the liquid motion in the region $\Omega_0^2/4g \ll 1$ has been derived. The model consists of a non-sloshing mass m_0 with the moment of inertia dyad

$$\tilde{I} = \begin{pmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{pmatrix}$$

where $I_{xx} = I_{yy}$, and of sloshing masses m_n connected with springs k_n. The introduction of damping was performed by adding linear dash pots to the sloshing masses. With the help of the Lagrange Equations the equations of motion of the mechanical model have been derived and yield the force equation, the moment equation and two times an infinite number of slosh equations.

Sincerely yours,

Helmut F. Bauer, Professor

HFB:ps

cc: AMSMI-RSD - 2
AMSMI-RSP - 1
AMSMI-IWD - 1

GEORGIA INSTITUTE OF TECHNOLOGY
School of Engineering Science and Mechanics

Monthly Cost and Performance Report # 13 Month of: September, 1969

Project Title: "Dynamics of Propellant in a Spinning Container and Description and a Mechanical Model:

Project No: B-903 Contract No: DAAH01-69-C-0296 Project Director: Dr. H. F. Bauer

Sponsor: U. S. Army Missile Command; Redstone Arsenal, Alabama

		FOR MONTH	TO DATE
Manhours Expended:	Project Director	115 *	881
	Graduate Assistants	115	1,473
Funds Expended:			
Personal Services		$1,443.18	$14,920.65
Material and Operating Expenses			
Travel		62.04 **	495.22
Computer Use Charges			
Overhead		822.61	8,504.76
Total Funds Expended		$2,327.83	$23,920.63
Percentage of Work Completed:		9%	96%

*Project Director performed part of this work on his own time, and was only paid for two weeks during the summer quarter, these two weeks being the last two weeks of September, 1969.

**Net amount. $12.72 adjustment on August trip.

Project Director

ENGINEERING COLLEGE

GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL OF ENGINEERING SCIENCE
AND MECHANICS

225 NORTH AVENUE, N.W.
ATLANTA, GEORGIA 30332

November 3, 1969

U. S. Army Missile Command
Redstone Arsenal, Alabama 35809

Attention: AMSMI-RSD

Subject: Monthly Progress Report #14
Contract #DAAH01 for period October 1-31, 1969
"Dynamics of Propellant in a Spinning Container and Description by a Mechanical Model."

Gentlemen:

During the subject period the mechanical model for the description of the liquid motion in a rotating container was completed. The mechanical values were determined. The formulas have been sent to Mr. Neale for numerical evaluation. Furthermore, some of the computer results have been discussed with Mr. Sofferis from the Army Computer Center, Redstone Arsenal, Alabama.

In addition, final report is now being written.

Very truly yours,

Helmut F. Bauer, Professor

HFB:mw

cc: AMSMI-RSD - 2
AMSMI-RSP - 1
AMSMI-IWD - 1

GEORGIA INSTITUTE OF TECHNOLOGY
School of Engineering Science and Mechanics

Monthly Cost and Performance Report # 14 Month of: October, 1969

Project Title: "Dynamics of Propellant in a Spinning Container and Description and a Mechanical Model"

Project No: B-903 **Contract No:** DAAH01-69-C-0296 **Project Director:** Dr. H. F. Bauer

Sponsor: U. S. Army Missile Command; Redstone Arsenal, Alabama

		FOR MONTH	TO DATE
Manhours Expended:	Project Director	56*	937
	Graduate Assistants	47	1,520
Funds Expended:			
Personal Services		$ 282.16	$15,202.81
Material and Operating Expenses			
Travel			495.22
Computer Use Charges			
Overhead		160.83	8,665.59
Total Funds Expended		$ 442.99	$24,363.62
Percentage of Work Completed:		2%	98%

*Work performed on his own time; not paid by this contract.

Project Director